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RESEARCH ARTICLE

Uncertainties in static nuclear properties due to pairing fit procedures within Skyrme-Hartree-Fock approach

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Abstract

Description of static nuclear properties is one of the main aims of theoretical nuclear physics studies. This work presents the uncertainties due to pairing fit procedure for three static nuclear quantities of rare-earth nuclei, namely the nuclear charge radius, electric quadrupole moment, and the moment of inertia. The Hartree-Fock (HF)-plus-pairing approach was employed with pairing correlations treated within the Bardeen-Cooper-Schrieffer (BCS) framework. The Skyrme SIII parametrization were chosen to approximate the effective nucleon-nucleon interaction while seniority force is used for the pairing interaction. In this work, two sets of pairing strengths obtained from different fit procedures were chosen. Calculated result shows that the moment of inertia is hugely dependent on variation of pairing strengths.

Keywords: Hartree-Fock, pairing, seniority force, rare earth, moment of inertia

INTRODUCTION

Currently, we are facing the Fourth Industrial Revolution whereby one of the key areas is clean and sustainable energy production. Nuclear physics has played a major role in energy production via nuclear fission process thus far. After the first introduction of commercial nuclear power plant in the 1950s, nuclear power has supplied about 11 % of the world electricity [1]. Nuclear fission energy is considered clean and is the world second largest low-carbon power supply [1]. In the meantime, scientists are working hard to realize the dream of building a nuclear fusion reactor that could potentially solve some problems faced by nuclear fission reactors.

In either nuclear fusion or fission applications for power generation, fundamental knowledge gained from theoretical and experimental studies is important. One example, in basic research would be able to provide useful information in determining the suitable materials for nuclear reactor applications through reaction cross-section data. This goal can be made possible through the present theoretical investigation. In the present work, a mean-field model is tested and benchmarked to some static nuclear properties.

The theoretical model used herein is based on the Hartree-Fock (HF)-plus-Bardeen-Cooper-Schrieffer (BCS) (HF+BCS) [2,3] mean-field approach. Recently, some studies on odd-mass nuclei have been performed within this mean-field approach on magnetic dipole moment [4], band-head energies and fission-barrier calculations of actinide nuclei [5,6], and preliminary studies in the rare earth region [7]. More recently, Ref. [8] explored different fit procedures in which the pairing strengths can be determined. This present research extends the study of Ref. [8] whereby preliminary investigations on the impact of different pairing strengths obtained through different fitting schemes are presented.

TECHNICAL DETAILS OF CALCULATION

The effective nucleon-nucleon interaction entering the HF framework is approximated by the phenomenological Skyrme interaction. Throughout this work, the SIII parametrization [9] is used. Seniority force is used to approximate the residual pairing interaction whereby the pairing strengths, G_q entering the pairing matrix element, g_q is given by

$$g_q = \frac{G_q}{11+N_q}. \qquad (1)$$

The sets of pairing strengths obtained from the fit of Ref. [8] and considered for this study are

- **Set A** – G_n = 16.0 MeV, G_p = 15.0 MeV
- **Set B** – G_n = 14.8 MeV, G_p = 12.4 MeV.

The main focus of this study is to investigate the impact of different values of pairing strengths on some static nuclear properties. Calculations are performed by fixing some input parameters such as (i) the proton and neutron numbers and (ii) corresponding basis parameters of the nucleus under study; (iii) type of Skyrme parametrization – chosen here to be of the SIII type, (iv) type of residual pairing interaction – chosen to be the seniority force, and (v) the intensity of residual pairing given in Set A or Set B above. Iterative calculations are then performed until convergence is achieved. The eigenfunction obtained from solving the Schrodinger-like equation at each iteration is used to calculate the expectation values of other quantities of interest.

One of the considered nuclear property is the charge radii. The mean square charge radius is given by

$$\langle \boldsymbol{r}^2 \rangle = \frac{\int_0^R \rho(r)\, \boldsymbol{r}^2\, dr}{\int_0^R \rho(r)\, dr}\,, \qquad (2)$$

whereby $\rho(\boldsymbol{r})$ is the proton density and R is the radius of the charge distribution. Most experimental studies on nuclear radii have been performed with results given in terms of the root mean square (rms) charge radius or its related quantities (see e.g. Ref. [10]).

Other than this, the other property that we are interested in is the intrinsic quadrupole moment, Q_{20} given by

$$Q_{20} = \int d^3r\, \rho(\boldsymbol{r})\,(2z^2 - r^2)\,, \qquad (3)$$

where $\rho(\boldsymbol{r})$ is the nucleon (or charge) density and $r^2 = x^2 + y^2$ is the radius in the $x - y$ plane.

Another property of interest is the moment of inertia which is known to be heavily affected by pairing correlations. Here, we seek to provide a quantitative assessment to the order of impact. The moment of inertia has been calculated herein using the Inglis-Belyaev's formula and then multiplied by a factor of 1.32 to account for the Thouless-Valatin terms. Further details on the calculation of the moment of inertia can be extracted from Ref [11].

RESULTS AND DISCUSSION

Nuclear charge radius

The results for the nuclear charge radii are tabulated in Table 1 for even-even nuclei and in Table 2 for odd-mass nuclei. Experimental data obtained from Ref. [12] and Ref. [10] are listed for comparison. It can be seen that both sets of calculations give rather good agreement with experimental data. Moreover, the variation in pairing strengths is seen to yield almost no effect on the charge radii with only a difference of less than 0.1 fm between data shown in Set A and Set B.

Table 1 Charge radii of some even-even nuclei defined as $r = \sqrt{\langle \hat{r}^2 \rangle}$. Experimental data were taken from Ref. [12] and Ref. [10]. All values are given in units of femtometer (fm).

Nucleus	Nuclear Radii (fm)			
	Set A	Set B	Exp. [12]	Exp. [10]
Yb-170	5.308	5.356	5.286	5.2853
Yb-172	5.303	5.380	5.301	5.2995
Yb-174	5.331	5.396	5.317	5.3108
Yb-176	5.341	5.412	5.321	5.3215
Yb-178	5.352	5.430	-	-
Hf-176	5.350	5.399	5.331	5.3286
Hf-178	5.359	5.415	5.338	5.3371
Hf-180	5.371	5.434	5.349	5.347
Hf-182	5.379	5.452	-	5.3516

Table 2 Charge radii of some odd-mass nuclei. Experimental data were taken from Ref. [10]. All values are given in units of femtometer (fm).

Nucleus	K^π	Nuclear Radii (fm)		
		Set A	Set B	Exp. [10]
Hf-177	$7/2^-$	5.356	5.408	5.3309
Hf-179	$9/2^+$	5.366	5.424	5.3408
Hf-181	$1/2^-$	5.377	5.443	-
Tm-169	$1/2^+$	5.294	5.349	5.349
Tm-171	$1/2^+$	5.311	5.373	5.331
Tm-173	$1/2^+$	5.319	5.388	5.338

Intrinsic electric quadrupole moment

Table 3 shows the calculated intrinsic quadrupole moment obtained from the two sets of pairing strengths. Set A pairing strengths yield a slightly better agreement to data with a difference of about 0.33 b. However, the discrepancies between results obtained using Set A and Set B are miniscule with only a small difference of about 0.14 b.

Table 3 Intrinsic electric quadrupole moment, Q_{20} for some ground-state even-even nuclei. Experimental data were taken from Ref. [13] for comparison with the value in parentheses representing the uncertainties in the last digits. All data are tabulated in units of barn (b).

Nucleus	Q_{20} (b)		
	Set A	Set B	Exp. [13]
^{162}Dy	7.420	7.487	7.33(8)
^{164}Dy	7.597	7.644	7.503(33)
^{166}Dy	7.724	7.765	-
^{168}Dy	7.796	7.903	-
^{168}Er	7.89	7.941	7.63(7)
^{170}Er	7.976	8.122	7.65(7)
^{172}Er	7.738	7.841	-

Moment of inertia

Table 4 shows the moment of inertia of some odd-mass nuclei obtained from the two considered sets of pairing strengths. The experimental moment of inertia has been derived from the energy of the first 2^+ and 4^+ excited states of the ground-state band using the expression

$$\mathcal{J} = \frac{4\hbar^2}{E(4^+) - 2E(2^+)}\,. \qquad (4)$$

Table 4 Moment of inertia of even-even nuclei obtained using the two sets of pairing strengths. All tabulated values are given in units of $\hbar^2$/MeV.

Nucleus	Moment of Inertia		
	Set A	Set B	Exp. [14]
^{156}Sm	41.208	51.879	40.846
^{158}Sm	41.536	52.677	42.239
^{160}Sm	44.487	58.840	43.716
^{160}Gd	39.694	52.747	40.816
^{162}Gd	44.578	62.419	42.918
^{164}Gd	42.953	56.058	41.973
^{162}Dy	38.053	51.095	38.355
^{164}Dy	43.337	61.384	41.908
^{166}Dy	41.321	53.170	39.859
^{168}Dy	43.997	60.275	40.646
^{168}Er	39.266	48.283	38.285
^{170}Er	42.371	47.248	38.854
^{172}Er	36.720	46.055	39.526

From the results, it is obvious that the moment of inertia is strongly affected by the change in pairing strengths. The calculated moment of inertia obtained using Set A reproduces experimental data within 3.35 $\hbar^2$/MeV while Set B reproduces experimental data within 22.296 $\hbar^2$/MeV.

CONCLUSION

In this paper, we present a preliminary study on the effect of pairing strengths on three different static nuclear properties. The two sets of pairing strengths used herein were obtained from different fit procedures. The first set (Set A) was obtained from a more robust fit as compared to second set (Set B). For some quantities such as charge radii and electric quadrupole moment, the change in pairing strengths do not cause substantial variation in the calculated results. However, quantities that are strongly dependent on pairing correlations such as moment of inertia would be strongly affected. In such cases, the proper fit procedure employed to obtain Set A is seen to play an important role in reproducing experimental data.

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